



## Advanced Topics in Numerical Analysis I Homework Assignment No. 10

(WS 2006/2007)

December 21, 2006

### Problem 37 (no correction)

Let  $\mathbb{GL}(n) := \{A \in \mathbb{R}^{n,n} : \det(A) \neq 0\}$ . Consider the function

$$f: \mathbb{GL}(n) \rightarrow \mathbb{GL}(n), \quad A \mapsto A^{-1}.$$

Compute the directional derivative

$$D_H f(A) := \lim_{t \rightarrow 0} \frac{1}{t} [f(A + tH) - f(A)]$$

for  $H \in \mathbb{GL}(n)$ .

### Problem 38 (to be handed in)

Consider the approximation of the exponential map by the sum

$$\exp(x) = \sum_{k=0}^N \frac{x^k}{k!} + \text{Error} \quad (1)$$

Compute approximative values of  $\exp(-5.5)$  with  $N = 3, 6, 9, \dots, 30 \dots$

- and formula (1).
- with  $\exp(-5.5) = 1/\exp(5.5)$  and formula (1).
- with  $\exp(-5.5) = (\exp(0.5))^{-11}$ .

Explain the results.

### Problem 39 (no correction)

Let  $\mathbb{S}_{>}^n := \{M \in \mathbb{R}^{n,n} : M \text{ symmetric positive definite}\}$ .

- Let  $A \in \mathbb{S}_{>}^n$ . Show, that there exists a unique  $W \in \mathbb{S}_{>}^n$  with  $W^2 = A$ .
- Let  $B \in \mathbb{R}^{n,n}$  regular. Show,  $B$  has a unique *polar decomposition*  $B = QU$  respectively  $B = VP$ , where  $Q, P \in \mathbb{R}^{n,n}$  orthogonal and  $U, V \in \mathbb{S}_{>}^n$ .

### Problem 40 (to be handed in)

Let  $A = (a_{ij}) \in \mathbb{R}^{n,n}$  a regular matrix. Proof the estimate

$$|\det A| \leq \prod_{j=1}^n \left( \sum_{i=1}^n |a_{ij}|^2 \right)^{1/2}$$

by using the QR-decomposition.

### Christmas problem

Find two numbers  $z_1, z_2$  with  $0 \leq z_1, z_2 \leq 100$ .

Peter knows the product  $(z_1 z_2)$  and Simon the sum  $(z_1 + z_2)$ . The following dialogue takes place.

Peter: "I do not know the numbers."

Simon: "I do not know the numbers, too."

Peter: "Thus, I know them."

Simon: "Thus, I know them, too."

Do you know the two numbers?

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Please hand in your homework problems (No. **38** and **40**) due **Thursday, January 11, 2006, 13:00h**. Put them in the slot marked „Numerische Mathematik I/II/III“ in the Math-Building (20.30), 2nd floor opposite room 112. Please print your name and registration number on your problems.

On **Thursday, January 11, 2006, 14:00-15:30 h** the problems will be discussed in the Neuer Hörsaal (Building 20.40).

Each Thursday a homework assignment will be handed out in the tutorial. The homework assignments are also available for download in the WWW:

<http://www.mathematik.uni-karlsruhe.de/ianm3/lehre/numana12006w> .