Universität Karlsruhe (TH)
Research University • founded 1825

Institute for Applied and Numerical Mathematics

Prof. Dr. Rudolf Scherer, Dipl.-Math. Wolfgang Müller

Advanced Topics in Numerical Analysis I
Homework Assignment No. 11

(WS 2006/2007) January 11, 2007

Problem 41 (to be handed in)

Consider a diagonalizable matrix $A \in \mathbb{R}^{n\times n}$ with eigenvalues

$$|\lambda_1| = |\lambda_2| > |\lambda_3| \geq \cdots \geq |\lambda_n| \quad \text{and} \quad \lambda_1 = -\lambda_2 \in \mathbb{R}.$$

a) For which initial vectors $z^{(0)}$ holds for the iterated vector $z^{(k+1)} = Az^{(k)}$, $k = 0, 1, 2, \ldots,$

$$\frac{z_j^{(2k+2)}}{z_j^{(2k)}} \to \lambda_1^2, \quad k \to \infty \; ?$$

Give detailed reasons for.

b) Compute for the matrix

$$A = \begin{pmatrix} 4.2 & -3.4 & 0.3 \\ 4.7 & -3.9 & 0.3 \\ -5.6 & 5.2 & 0.1 \end{pmatrix}$$

and the initial vector $z^{(0)} = (0.2,\ 0.4,\ 0.6)^T$ the iterated vectors $z^{(i)}$, $i = 1, \ldots, 6$. Compute approximations to the two dominant eigenvalues with opposite sign using $z^{(0)}, \ldots, z^{(6)}$.

Problem 42 (to be handed in)

Compute approximations to the eigenvalues of the matrix

$$A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$$

by applying the inverse iteration of Wielandt to the shifted matrix $A_s := A - sI$ using Rayleigh quotients. For $s = 0$, $s = -1.5$ and $s = 3.5$ perform three iteration steps each, with the initial vector $z^{(0)} = (1, 0)^T$.

Problem 43 (no correction)

Consider the matrix

$$A = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 3 & 1 \\ 0 & 1 & 4 \end{pmatrix}.$$

Perform two steps of the QR method and determine approximations to the eigenvalues of A.

Problem 44 (no correction)

For the matrix $A \in \mathbb{R}^{n\times n}$ the characteristic polynomial can be computed using a Krylov method.

With the initial vector $x \in \mathbb{R}^n$ the iterated vectors

$$x^{(0)} := x,\ x^{(1)} := Ax^{(0)}, \ldots,\ x^{(n)} := Ax^{(n-1)}$$

are computed.

a) Show:
If $x^{(0)}, \ldots, x^{(n-1)}$ are linearly independent then the vector $x^{(n)}$ is given by the linear combination

$$-(-1)^n x^{(n)} = \sum_{j=0}^{n-1} a_j x^{(j)}$$

where the coefficients $a_0, \ldots, a_{n-1}$ define the characterisic polynomial φ_A of the matrix A,

$$\varphi_A(t) = a_0 + a_1 t + \cdots + a_{n-1}t^{n-1} + (-1)^n t^n.$$

b) Using this method and the initial vector $x = (1, 0, 0)^T$ compute the characteristic polynomial φ_B of the matrix

$$B = \begin{pmatrix} 3 & 2 & 0 \\ 0 & 1 & 4 \\ 2 & 1 & 4 \end{pmatrix}.$$

Please hand in your homework problems (No. **41** and **42**) due **Thursday**, **January 18, 2007**, **13:00h**. Put them in the slot marked „Numerische Mathematik I/II/III“ in the Math-Building (20.30), 2nd floor opposite room 112. Please print your name and registration number on your problems.

On **Thursday**, **January 18, 2007**, **14:00-15:30 h** the problems will be discussed in the Neuer Hörsaal (Building 20.40).

Each Thursday a homework assignment will be handed out in the tutorial. The homework assignments are also available for download in the WWW:

http://www.mathematik.uni-karlsruhe.de/ianm3/lehre/numana12006w .