Universität Karlsruhe (TH)
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Institute for Applied and Numerical Mathematics

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Advanced Topics in Numerical Analysis I
Homework Assignment No. 12

(WS 2006/2007) January 18, 2007

Problem 45 (to be handed in)

a) For the matrix

$$A_1 = \begin{pmatrix} 2 & a \\ \varepsilon & 1 \end{pmatrix}, \varepsilon > 0, a \in \mathbb{R}^+$$

perform one step of the QR method

i) without shift,

ii) with shift $s = 1$ $(= a_{22}^{(1)})$.

b) Discuss the order of magnitude of $a_{21}^{(2)}$ in the resulted matrix A_2 for $\varepsilon \to 0$. Consider both cases i) and ii) for fixed a and for the symmetric case $a = \varepsilon$.

Problem 46 (no correction)

Let $A \in \mathbb{R}^{n \times n}$ and $\lambda_1, \ldots, \lambda_s$ be the distinct eigenvalues of A. The minimal polynomial ψ of A is of degree r $(r \geq s)$. Show:

a) For every $x \in \mathbb{R}^n$ the iterated vectors $A^i x$, $i = 0, 1, \ldots, r$ are linearly dependent.

b) If $x \in \mathbb{R}^n$ is of the form $x = u_1 + \cdots + u_s$, where u_j is an eigenvector with respect to the eigenvalue λ_j, $j = 1, \ldots, s$, then the iterated vectors $A^i x$ for $i = 0, 1, \ldots, s-1$ are linearly independent.

Problem 47 (no correction)

Consider the matrix

$$A(\varepsilon) = \begin{pmatrix} 1 + \varepsilon\cos(2/\varepsilon) & -\varepsilon\sin(2/\varepsilon) \\ -\varepsilon\sin(2/\varepsilon) & 1 - \varepsilon\cos(2/\varepsilon) \end{pmatrix}.$$

a) Determine the eigenvalues $\lambda_i(\varepsilon)$ and the eigenvectors $v_i(\varepsilon)$ of the matrix $A(\varepsilon)$.

b) What happens with $A(\varepsilon)$, $\lambda_i(\varepsilon)$ and $v_i(\varepsilon)$ for $\varepsilon \to 0$?

Problem 48 (to be handed in)

a) For the initial value problem

$$y' = x - x^3, y(0) = 0,$$

with the solution u in the interval $[0, b]$, $b > 0$, we seek an approximate solution u_h on the grid

$$I_h = \{x_m = mh,\ m = 0, 1, \ldots, N,\ h = \frac{b}{N}\}$$

using the Euler polygon method.

Calculate $u_h(x_m)$ and $e_h(x_m) := u(x_m) - u_h(x_m)$ explicitly and show that $\lim_{h \to 0} e_h(b) = 0$.

b) Compute for the initial value problem

$$y'' - y = 0,\ y(0) = 1,\ y'(0) = 0$$

approximations to the solution $u(1)$ and its derivative $u'(1)$ using the Euler polygon method with stepsizes $h = \frac{1}{2}$ and $h = \frac{1}{4}$.

Please hand in your homework problems (No. **45** and **48**) due **Thursday**, **January 25, 2007**, **13:00h**. Put them in the slot marked „Numerische Mathematik I/II/III“ in the Math-Building (20.30), 2nd floor opposite room 112. Please print your name and registration number on your problems.

On **Thursday**, **January 25, 2007**, **14:00-15:30 h** the problems will be discussed in the Neuer Hörsaal (Building 20.40).

Each Thursday a homework assignment will be handed out in the tutorial. The homework assignments are also available for download in the WWW:

http://www.mathematik.uni-karlsruhe.de/ianm3/lehre/numana12006w .