Universität Karlsruhe (TH)
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Institute for Applied and Numerical Mathematics

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Advanced Topics in Numerical Analysis I
Homework Assignment No. 13

(WS 2006/2007) January 25, 2007

Problem 49 (to be handed in)

Observe the numerical approximation of the initial value problem

$$\begin{bmatrix} y_1' \\ y_2' \end{bmatrix} = \begin{bmatrix} y_2 \\ -y_1 \end{bmatrix}, \qquad y_1(0) = 0, \qquad y_2(0) = 1 \; .$$

a) Show: the first component of the approximated solution $u_h(x_m)$ satisfies a three term recursion formula for the forward and backward Euler method.

b) For both methods, give an explicit formula for the first component of the approximated solution $u_h(x_m)$. Use a suitable ansatz for the difference equations of a).

c) The analytic solution of the initial value problem is bounded for $x \in [0, b]$ with arbitrary b. Check whether or not the discrete methods fulfill this property, too.

Problem 50 (no correction)

Let f and g be continuous in the rectangle $R := [a, b] \times [-c, c]$ (c sufficiently large),

$$|f(x,y)| \le K \; , \quad (x,y) \in R \; ,$$
$$|f(x,y) - g(x,y)| \le \varepsilon \; , \quad (x,y) \in R \; .$$

Moreover f satisfies a Lipschitz condition with respect to y and the constant $L > 0$. u resp. $\tilde{u}$ is the solution of the initial value problem

$$y' = f(x,y), \quad y(x_0) = y_0 \quad \text{resp.} \quad y(\tilde{x}_0) = \tilde{y}_0, \quad a = x_0 \le \tilde{x}_0 \le b$$

and v the solution of the initial value problem

$$y' = g(x,y), \quad y(x_0) = y_0.$$

Proof for $x \in [a, b]$ the following estimates

$$|u(x) - \tilde{u}(x)| \le (|y_0 - \tilde{y}_0| + K|x_0 - \tilde{x}_0|) e^{L|x - x_0|},$$
$$|u(x) - v(x)| \le \frac{\varepsilon}{L}(e^{L|x - x_0|} - 1).$$

Problem 51 (no correction)

Compute for $n = 0, 1, 2, \ldots$ the Picard serie

$$\phi_{n+1}(x) = y_0 + \int_a^x f(t, \phi_n(t)) \, dt \; , \quad \phi_0(x) \equiv 0$$

for the initial value problem

$$y' = \lambda y, \quad x \in [0, b], \quad y(0) = 1 =: y_0 \; .$$

Discuss the numerical realization of this method.

Problem 52 (to be handed in)

Let $f \in C([a, b] \times \mathbb{R}^m, \mathbb{R}^m)$. Observe the initial value problem

$$y' = f(x, y(x)), \quad y(a) = y_0, \qquad \text{(IVP 1)}$$

The transformation $z := Qy$ with $Q \in \mathbb{R}^{m,m}$ regular, carries (IVP 1) over in

$$z' = Qf(x, Q^{-1}z), \quad z(a) = Qy_0 \; . \qquad \text{(IVP 2)}$$

a) Give reasons for the above formula. Or give an example.

b) Show: Each explicit Runge–Kutta method maintains this transformation behavior, i.e. If y_h is the approximated solution of (IVP 1), then $z_h = Qy_h$ is the approximated solution of (IVP 2).

Please hand in your homework problems (No. **49** and **52**) due **Thursday**, **February 1, 2007**, **13:00h**. Put them in the slot marked „Numerische Mathematik I/II/III" in the Math-Building (20.30), 2nd floor opposite room 112. Please print your name and registration number on your problems.

On **Thursday**, **February 1, 2007**, **14:00-15:30 h** the problems will be discussed in the Neuer Hörsaal (Building 20.40).

Each Thursday a homework assignment will be handed out in the tutorial. The homework assignments are also available for download in the WWW:

http://www.mathematik.uni-karlsruhe.de/ianm3/lehre/numana12006w .