Universität Karlsruhe (TH)
Research University • founded 1825

Institute for Applied and Numerical Mathematics

Prof. Dr. Rudolf Scherer, Dipl.-Math. Wolfgang Müller

Advanced Topics in Numerical Analysis I
Homework Assignment No. 15

(WS 2006/2007) February 8, 2007

Problem 57 (no correction)

In an implicit linear k-step method an approximation y_{m+k} to $u(x_{m+k})$ with initial value $y^{(0)}_{m+k}$ is computed using the following method:

$$y^{(i+1)}_{m+k} = y_{m+k-1} + h\Big(\beta_k f(x_{m+k}, y^{(i)}_{m+k}) + \sum_{j=0}^{k-1} \beta_j f_{m+j}\Big), \quad i = 0, 1, 2, \ldots$$

Show that for $f \in F_1(I)$ there exists an $h_0 > 0$ such that for all h with $|h| \le h_0$ the series $\{y^{(i)}_{m+k}\}$ converges.

Problem 58 (no correction)

Consider the 3-step method

$$y_{m+3} + \alpha(y_{m+2} - y_{m+1}) - y_m = \frac{h}{2}(3+\alpha)(f_{m+2} + f_{m+1}), \quad \alpha \in \mathbb{R}.$$

a) Determine the values α, for which the characteristic polynomial $\rho(\xi)$ satisfies the root condition.

b) Show that there exists an α for which the order of consistency is $p = 4$ whereas the order of consistency is $p = 2$ if $\rho(\xi)$ satisfies the root condition.

Problem 59 (no correction)

Given are an explicit and an implicit linear multistep method that are used in Predictor-Corrector form:

$$\begin{aligned} P: \quad & y^{(0)}_{m+1} && := y_{m-1} + 2hf_m, \\ E: \quad & f^{(0)}_{m+1} && := f(x_{m+1}, y^{(0)}_{m+1}), \\ K: \quad & y_{m+1} && := y_m + \tfrac{1}{2}h(f^{(0)}_{m+1} + f_m). \end{aligned}$$

a) Examine the explicit and the implicit method with regard to their order of consistency and show that the local discretization error is

$$\tau(x,y,h) = \frac{h^2}{3}u^{(3)}(\xi) \quad \text{resp.} \quad \tau(x,y,h) = -\frac{h^2}{12}u^{(3)}(\xi).$$

b) Apply the Predictor-Corrector method to the initial value problem $y' = 2xy, y(0) = 1$ in the interval $[0, 2]$ with the stepsize $h = \frac{1}{4}$ and the initial values $y_0 = 1, y_1 = 1.0645$ and determine $u_h(1)$.

On **Thursday**, **February 15, 2007**, **14:00-15:30 h** the problems will be discussed in the Neuer Hörsaal (Building 20.40).