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Institute for Applied and Numerical Mathematics

Prof. Dr. Rudolf Scherer, Dipl.-Math. Wolfgang Müller

Advanced Topics in Numerical Analysis I
Programming Exercise No. 1

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Programming exercise 1

A function $f \in C[a,b]$ is to be interpolated with respect to the nodes $x_j = \frac{2\pi j}{n}$ for $j = 0, \ldots, n-1$ with $n = 2^p$ $(m = \frac{n}{2})$ by a trigonometric polynomial

$$t(x) = \frac{a_0}{2} + \sum_{\nu=1}^{m-1} \left(a_\nu \cos(\nu x) + b_\nu \sin(\nu x)\right) + a_m \cos(mx) \ .$$

Write a computer program that

a) computes the fast Fourier transformation (**FFT**).

b) computes the inverse Fourier transformation (**IFFT**).

c) computes the coefficients $a_0, \ldots, a_m$ and $b_1, \ldots, b_{m-1}$ by the function call **TIP**(**IFFT**(y)).

As a starting template you can download the file FFT.C from the web site of the lecture.

You can check and debug your program with the following table.

j	0	1	2	3	4	5	6	7
x_j	0	$\pi/4$	$\pi/2$	$3\pi/4$	π	$5\pi/4$	$3\pi/2$	$7\pi/4$
y_j	1	8	-1	4	1	8	-1	4
$(\textbf{FFT}(y))_j$	24	0	$4+8i$	0	-24	0	$4-8i$	0
$(\textbf{IFFT}(y))_j$	3	0	$0.5-i$	0	-3	0	$0.5+i$	0
a_j	6	0	1	0	-3	–	–	–
b_j	–	0	2	0	–	–	–	–

Compute with your program $(\textbf{FFT}(y))_{100}$, $(\textbf{IFFT}(y))_{10}$, a_{30} and b_6 for $n = 2^{18}$ and $f(x) = x(x-2\pi)\exp(x/10)$. How does computing time change, if you choose $n = 2^{19}$?

Algorithm for the function $w =$ FFT(y).

(S0) Let $y \in \mathbb{C}^n$ and $m = n/2$.

(S1) If $n = 1$: result is $w = y$. STOP.

(S2) Set $z = \exp(2\pi i/n)$.

(S3) Generate vectors $\tilde{y}_j = y_{2j}$ and $\hat{y}_j = y_{2j+1}$ for $j = 0, ..., m-1$. Compute $p =$ **FFT**$(\tilde{y})$ and $q =$ **FFT**$(\hat{y})$.

(S4) Compute $r = Dq$ with $D = \mathrm{diag}(1, z, \ldots, z^{m-1})$.

(S5) Result is $w = \left[(p+r)^T, (p-r)^T\right]^T$. STOP.

Algorithm for the function $c =$ IFFT(y).

(S0) Let $y \in \mathbb{C}^n$.

(S1) Generate vector $\bar{y} = [y_0, y_{n-1}, y_{n-2}, \ldots, y_1]^T$.

(S2) Result is $c =$ **FFT**$(\bar{y})/n$. STOP.

Algorithm for the function $[a,b] =$ TIP(c).

(S0) Let $c \in \mathbb{C}^n$ and $m = n/2$.

(S1) Set $c_n = c_0$.

(S2) Compute $a_\nu = c_\nu + c_{n-\nu}$ and $b_\nu = i(c_\nu - c_{n-\nu})$ for $\nu = 0, ..., m-1$.

(S3) Set $a_m = c_m$. STOP.

Please hand in your programming exercise due **Wednesday, November 22, 2006** to the tutor Mr. Maurer in the computing center, K-Pool (room 114a), 14:00-17:00h.

The programming exercises will be handed out in the lecture on Wednesday. And are also available for download in the WWW:

http://www.mathematik.uni-karlsruhe.de/ianm3/lehre/numana12006w .