Universität Karlsruhe (TH)
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Institute for Applied and Numerical Mathematics

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Advanced Topics in Numerical Analysis I
Programming Exercise No. 4

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Prgramming exercise 4

Write a computer program that uses the Jacobi method to solve the eigenvalue problem

$$Ax = \lambda x$$

for symmetric matrices $A \in \mathbb{R}^{n\times n}$.

Applying Jacobi rotations J_k we get a series of orthogonal similar matrices $A^{(k)}$,

$$A^{(k)} = J_k A^{(k-1)} J_k^{-1} = (a_{ij}^{(k)}), \quad A^{(0)} = (a_{ij}^{(0)}) = A$$

with the same eigenvalues. The idea of the method is to choose the rotations J_k such that the Schur norm of the non-diagonal elements of $A^{(k)}$,

$$M(A^{(k)}) = \left(\sum_{i,j=1, i\neq j}^{n} |a_{ij}^{(k)}|^2 \right)^{1/2}$$

decreases step by step. So the diagonal elements of $A^{(k)}$ are approximations to the eigenvalues of A.

Given $i \neq j \in \{1, \ldots, n\}$ choose J_k such that $a_{ij}^{(k)} = a_{ji}^{(k)} = 0$ with an rotation angle θ, $|\theta| \leq \frac{\pi}{4}$.

Take into account that only the rows and columns i and j of $A^{(k-1)}$ change.

Consider the following two possibilities for the choice of the indices (i, j):

a) Classical method:

   In the k-th step i and j are chosen such that $a_{ij}^{(k-1)}$ is the largest non-diagonal element in absolute value of $A^{(k-1)}$.

b) Cyclic method:

   Choose (i, j) in the following way:

$$(2,1); (3,1), (3,2); (4,1), \ldots, (n, n-1); (2,1), \ldots$$

Check the value of $M(A^{(k)})$ after each $N = \frac{n(n-1)}{2}$ steps and stop the iteration, if

$$M(A^{(k)}) < 10^{-4} N_e(A)$$

($N_e(A) = \left(\sum_{i,j=1}^{n} |a_{ij}|^2 \right)^{1/2}$ the Schur norm of A).

Print out the value $M(A^{(k)})$ for $k = N, 2N, \ldots$ and the final approximations to the eigenvalues of A.

Compute with your programm the following examples:

$$A_1 = \begin{pmatrix} 2 & -1 & & 0 \\ -1 & 2 & \ddots & \\ & \ddots & \ddots & -1 \\ 0 & & -1 & 2 \end{pmatrix} \in \mathbb{R}^{20\times 20},$$

$$\sigma(A_1) = \{2(1 - \cos\frac{k\pi}{21}),\ 1 \leq k \leq 20\}.$$

$$A_2 = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 1 & 3 & 6 & 10 \\ 1 & 4 & 10 & 20 \end{pmatrix},$$

$$\sigma(A_2) = \{0.0380,\ 0.4538,\ 2.2034,\ 26.3047\}.$$

Change of date

On **17.01.2007 and 24.01.2007** the programming assistance will be at

A-Pool (room -112): 14:00-15:45 h,
K-Pool (room 114a): 16:00-17:00 h.

As from **31.01.2007** it will be at
K-Pool (room 114a): 14:00-17:00 h.

Please hand in your programming exercise due **Wednesday**, **January 31, 2007** in computing center. Each 2nd Wednesday a programming exercise will be handed out in the tutorial. The programming exercises are also available for download in the WWW:

http://www.mathematik.uni-karlsruhe.de/ianm3/lehre/numana12006w .