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Institute for Applied and Numerical Mathematics

Prof. Dr. Rudolf Scherer, Dipl.-Math. Wolfgang Müller

Advanced Topics in Numerical Analysis I
Programming Exercise No. 5

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Prgramming exercise 5

The orbit of a satellite in the gravitation field of moon and earth.

Write a program to compute the solution of the initial value problem

$$
\begin{aligned}
y_1' &= y_3 \\
y_2' &= y_4 \\
y_3' &= 2y_4 + y_1 - \hat{\mu}(y_1+\mu)/r_1(y) - \mu(y_1-\hat{\mu})/r_2(y) \\
y_4' &= -2y_3 + y_2 - \hat{\mu}y_2/r_1(y) - \mu y_2/r_2(y) \\
\tilde{r}_1(y) &= (y_1+\mu)^2 + y_2^2, \quad r_1(y) = \tilde{r}_1(y)\sqrt{\tilde{r}_1(y)} \\
\tilde{r}_2(y) &= (y_1-\hat{\mu})^2 + y_2^2, \quad r_2(y) = \tilde{r}_2(y)\sqrt{\tilde{r}_2(y)} \\
\mu &= 0.012277471 \\
\hat{\mu} &= 1-\mu \\
y_1^0 &= 0.994 \\
y_2^0 &= 0 \\
y_3^0 &= 0 \\
y_4^0 &= -2.0015851063790825
\end{aligned}
$$

in the interval $[0,b] = [0, 17.0652165601579622]$ with an embedded Runge–Kutta method which is given by the following scheme

$$
\left(\begin{array}{c|ccccc|cc}
0 & 0 & & & & & \frac{1}{6} & \frac{1}{6} \\
\frac{1}{2} & \frac{1}{2} & 0 & & & & \frac{2}{6} & \frac{2}{6} \\
\frac{1}{2} & 0 & \frac{1}{2} & 0 & & & \frac{2}{6} & \frac{2}{6} \\
1 & 0 & 0 & 1 & 0 & & \frac{1}{6} & 0 \\
1 & \frac{1}{6} & \frac{2}{6} & \frac{2}{6} & \frac{1}{6} & 0 & 0 & \frac{1}{6}
\end{array}\right).
$$

The last column describes a one-step method of order three. The other is of order four.

Implement a step-size control algorithm.

Algorithm for step-size control.

(S0) Let be given two Runge–Kutta methods $\phi, \hat{\phi}$ with order p respectively $p-1$. Choose initial values $x_0 = 0$, $y^0 \in \mathbb{R}^n$ and starting step-size $h > 0$. Moreover, choose a tolerance $\varepsilon > 0$ and a safety multiplier $0 < \vartheta < 1$. Set $m = 0$.

(S1) Until $x_m < b$ compute u_{m+1} and $\hat{u}_{m+1}$ with ϕ respectively $\hat{\phi}$.

(S2) Compute an error estimate $|\eta_{m+1}| = |\hat{u}_{m+1} - u_{m+1}|$.

(S3) If $|\eta_{m+1}| < \varepsilon$: Set $x_{m+1} = x_m + h$ and $m = m+1$.

(S4) Compute new step-size $h = \vartheta h\big(\frac{\varepsilon}{|\eta_{m+1}|}\big)^{1/(p-1)}$ and go to (S1).

For the tolerance choose the values $\varepsilon = 10^{-2}, 10^{-3}, 10^{-5}, 10^{-10}$. With a starting step-size of $h = 10^{-4}$. For the safety multiplier choose $\vartheta = 0.8$. Summarize your results in a tabular which includes $|u_h(b) - u^0|$, the number of steps N and the number of function evaluations for every ε.

Date of programming assistance

From now on the programming assistance will be on wednesday at K-Pool (room 114a): 14:00-17:00h.

Please hand in your programming exercise due **Wednesday**, **February 14, 2007** in computing center. Each 2nd Wednesday a programming exercise will be handed out in the tutorial. The programming exercises are also available for download in the WWW:

http://www.mathematik.uni-karlsruhe.de/ianm3/lehre/numana12006w .