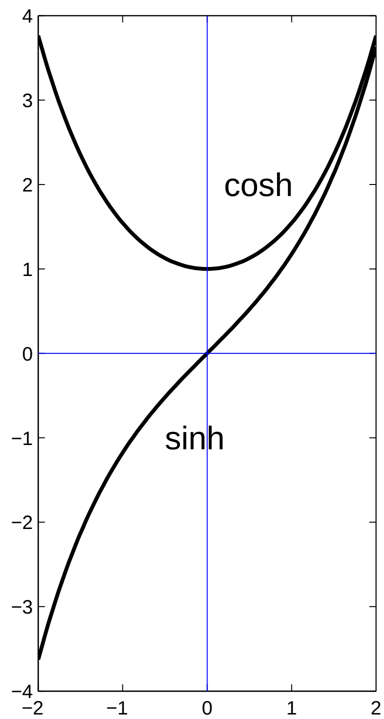
Lösung zu 3.4: a)



b) Einsetzen der Definition führt auf

$$\begin{aligned}\cosh^2 x - \sinh^2 x &= \frac{1}{4}\left[(e^x + e^{-x})^2 - (e^x - e^{-x})^2\right] \\ &= \frac{1}{4}\left[e^{2x} + 2e^x e^{-x} + e^{-2x} - e^{2x} + 2e^x e^{-x} - e^{-2x}\right] = \frac{1}{4} 4e^x e^{-x} = 1\end{aligned}$$

Weiter gilt

$$\begin{aligned}&\cosh x \, \cosh y + \sinh x \, \sinh y \\ &= \frac{1}{4}\left[(e^x + e^{-x})(e^y + e^{-y}) + (e^x - e^{-x})(e^y - e^{-y})\right] \\ &= \frac{1}{4}\left[e^x\, e^y + e^{-x}\, e^y + e^x\, e^{-y} + e^{-x}\, e^{-y} + e^x\, e^y - e^{-x}\, e^y - e^x\, e^{-y} + e^{-x}\, e^{-y}\right] \\ &= \frac{1}{4}\left[2e^x\, e^y + 2e^{-x}\, e^{-y}\right] = \frac{1}{2}\left[e^{x+y} + e^{-(x+y)}\right] = \cosh(x+y).\end{aligned}$$

c) Wir berechnen

$$\begin{aligned}\sinh(\ln(x + \sqrt{1+x^2})) &= \frac{1}{2}(e^{\ln(x+\sqrt{1+x^2})} - e^{-\ln(x+\sqrt{1+x^2})}) \\ &= \frac{1}{2}((x + \sqrt{1+x^2}) - \frac{1}{x + \sqrt{1+x^2}}) \\ &= \frac{x^2 + 1 + x^2 + 2x\sqrt{1+x^2} - 1}{2(x + \sqrt{1+x^2})} = \frac{2x(x + \sqrt{1+x^2})}{2(x + \sqrt{1+x^2})} = x\end{aligned}$$

und

$$
\begin{aligned}
\ln(\sinh x + \sqrt{1+\sinh^2 x}) &= \ln\left(\frac{e^x - e^{-x}}{2} + \sqrt{1 + \frac{1}{4}(e^x - e^{-x})^2}\right) \\
&= \ln\left(\frac{e^x - e^{-x}}{2} + \sqrt{\frac{1}{4}e^{2x} + \frac{1}{2} + \frac{1}{4}e^{-2x}}\right) \\
&= \ln\left(\frac{e^x - e^{-x}}{2} + \sqrt{\frac{1}{4}(e^x + e^{-x})^2}\right) \\
&= \ln\left(\frac{e^x - e^{-x}}{2} + \frac{1}{2}(e^x + e^{-x})\right) = \ln e^x = x
\end{aligned}
$$

Also ist $\operatorname{arsinh} x = \ln(x + \sqrt{1+x^2})$.