Karlsruher Institut für Technologie

Course Catalog

Winter term 2024/25

KARLSRUHE INSTITUTE FOR TECHNOLOGY

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1. KIT Department of Mathematics

1.1. Courses and Seminars of the International Program

0104800
4 SWS

Functional Analysis
🗣 On-Site
Mon 15:45-17:15 30.41 Chemie-Hörsaal Nr. 3 (HS3) from 10/21 until 02/10
Wed 11:30-13:00 20.40 Neuer Hörsaal Architektur from 10/23 until 02/12

Lecture (V)
Reichel, Wolfgang

Content

The lecture deals with Banach and Hilbert spaces and the linear operators on these spaces. Typical examples are spaces of continuous or integrable functions, and linear operators on these spaces occur in the study of integral and differential equations. The development of functional analysis in the 20th century contributed significantly to the modern theory of differential equations. Today, functional analysis is a fundamental discipline of modern analysis and is widely used, for example, in the theory of partial differential equations, numerical mathematics, mathematical physics and many other areas of application.

Topics of the lecture:
basic properties and examples of metric spaces and Banach spaces, continuous linear operators on Banach spaces, uniform boundedness principle, homomorphism theorem, Hilbert spaces, orthonormal bases, Sobolev spaces, dual spaces, Hahn-Banach theorem, weak convergence, Banach-Alaoglu theorem, reflexivity, compact linear operators

The contents of the basic lectures Analysis 1-3 and Linear Algebra 1+2 are assumed.

Literature

- D. Werner: Funktionalanalysis.

- H.W. Alt: Lineare Funktionalanalysis.
- H. Brezis: Functional Analysis, Sobolev Spaces and Partial Differential Equations.
- J.B. Conway: A Course in Functional Analysis.
- M. Reed, B. Simon: Functional Analysis.
- W. Rudin: Functional Analysis.
- A.E. Taylor, D.C. Lay: Introduction to Functional Analysis.
- J. Wloka: Funktionalanalysis und Anwendungen.

0104810
2 SWS

Tutorial for 0104800 (Functional Analysis)
🗣 On-Site
Fri 11:30-13:00 20.30 SR 1.067 from 10/25 until 02/14

Practice (Ü)
Reichel, Wolfgang

0105360
4 SWS

Nonlinear Maxwell equations
Mon 09:45-11:15 20.30 SR 2.067 from 10/21 until 02/10
Tue 09:45-11:15 20.30 SR 2.067 from 10/22 until 02/11

Lecture (V)
Schnaubelt, Roland

Content

The Maxwell equations are the fundamental laws governing electro-magnetic phenomena. Constitutive relations describe the interaction between the fields and the material by determining the polarization or magnetization. We study nonlinear relations focussing on an instantaneous material response. The Maxwell system then becomes a quasilinear hyperbolic system. We concentrate on the full space case, as problems on domains with boundary conditions (or interface problems) are much more demanding. First, we establish local wellposedness for initial values in H^3 by energy methods involving linearisation, apriori estimates and regularisation. We will further study blow-up phenomena despite energy preservation. In the second main part we investigate dispersive behavior in terms of Strichartz inequalities and improve the wellposedness theory. These theorems were shown only very recently. We also intend to give an outlook to further recent results on the longtime behavior and problems on domains.

The lectures requires knowledge in functional analysis and basics of Sobolev spaces. Further tools will be discussed in the lectures, possibly without giving complete proofs.

0105370 2 SWS	**Tutorial for 0105360 (Nonlinear Maxwell equations)** Thu 15:45-17:15 20.30 SR 3.069 from 10/24 until 02/13	**Practice (Ü)** Schnaubelt, Roland
0100002 2 SWS	**Advanced Methods in Nonlinear PDEs** Wed 09:45-11:15 20.30 SR 2.058 from 10/23 until 02/12	**Lecture (V)** de Rijk, Björn

Content

Nonlinear partial differential equations describing physical phenomena are often complex, making their qualitative and quantitative analysis challenging. Amplitude or modulation equations, such as the Ginzburg-Landau equation, the Korteweg-de Vries equation, and the nonlinear Schrödinger equation, play an important role in capturing the critical dynamics of spatially extended dissipative or conservative physical models. Mathematical theorems demonstrate that these well-understood asymptotic models accurately predict the behavior of the original system on sufficiently long time scales. Examples of regimes which can be described in such a way include pattern-forming systems close to their first instability, the long-wave limit of the water wave problem, and highly oscillatory regimes in nonlinear optics.

In the first part of this course, we develop several methods to rigorously justify approximations of complex physical systems by amplitude or modulation equations. Relevant tools include Fourier analysis, energy estimates, semigroup theory, mode filters, and normal form transformations. Often, amplitude or modulation equations admit special solutions, such as periodic patterns, solitary waves, or traveling (modulating) fronts. While approximation results yield solutions of the original system that are close to these special solutions, they are insufficient to conclude that such special solutions exist in the original system as well. In the second part of this course, we focus on techniques, such as Lyapunov-Schmidt reduction, spatial dynamics, and center manifold reduction, to construct these special solutions in the original system.

We will roughly follow Chapter 10-13 from the book:

Schneider, Guido; Uecker, Hannes. Nonlinear PDEs. A dynamical systems approach. Graduate Studies in Mathematics, 182. American Mathematical Society, Providence, RI, 2017. xiii+575 pp. ISBN: 978-1-4704-3613-1

The course requires a solid background in analysis (Analysis 1-3). It is strongly recommended to have attended a lecture in functional analysis and on evolution equations.

0156400 4 SWS English	**Harmonic Analysis** Tue 08:00-09:30 20.30 SR 2.066 from 10/22 until 02/11 Thu 09:45-11:15 20.30 SR 2.066 from 10/24 until 02/13	**Lecture (V)** Frey, Dorothee

Content

- Fourier series
- Fourier transform on L^1 and L^2
- Tempered distributions and their Fourier transform
- Explicit solutions of the Heat-, Schrödinger- and Wave equation on R^n
- the Hilbert transform
- the interpolation theorem of Marcinkiewicz
- Singular integral operators
- the Fourier multiplier theorem of Mihlin

0156410 2 SWS	**Tutorial for 0156400 (Harmonic Analysis)** Wed 15:45-17:15 20.30 Seminarraum -1.025 (UG) from 10/23 until 02/12	**Practice (Ü)** Frey, Dorothee

0165900
3 SWS

Introduction to Fluid Mechanics — **Lecture (V)** — Liao, Xian

Tue 14:00-15:30 20.30 Seminarraum 0.019 from 10/22 until 02/11

Content

The motion of fluids is ubiquitous in our daily lives and can be described mathematically by two fundamental partial differential equations: the Euler equations and the Navier-Stokes equations. Although many intriguing open problems remain in the mathematical theory of solutions to these models, some well-established theories address fundamental problems concerning the existence and uniqueness of solutions. The mathematical analysis is highly sensitive to the types and parameters of the models; for example, the solutions of the Euler equations for inviscid fluids and those of the Navier-Stokes equations for viscous fluids exhibit notably different analytical properties.

We investigate both incompressible models (e.g., describing the motion of water) and compressible models (e.g., describing the motion of gases). By the end of the lecture, the audience will understand the distinct characteristics of these two types of models. Topics include the existence and uniqueness of weak and strong solutions, as well as the free boundary problem (if time permits).

The basic mathematical theory aligns with the lecture given in the summer semester of 2023 (https://www.math.kit.edu/iana2/edu/introfluidmech2023s/en), and we will also include compressible models in the winter semester of 2024/2025.

Prerequisites: Analysis 1-3, Functional Analysis

0165910
1 SWS

Tutorial for 0165900 (Introduction to Fluid Mechanics) — **Practice (Ü)** — Liao, Xian

Fri 09:45-11:15 20.30 SR 2.066 from 10/25 until 02/14

0110300
4 SWS

Finite Element Methods — **Lecture (V)** — Maier, Roland

Mon 11:30-13:00 20.30 SR 2.058 from 10/21 until 02/10
Tue 08:00-09:30 20.30 Seminarraum -1.025 (UG) from 10/22 until 02/11

Content

This course is about numerically solving elliptic boundary value problems using the finite element method. We will introduce necessary basic definitions and then discuss the method in its various aspects. In particular, we will study grid generation, error estimates, and the practical realization of the approach.

0110310
2 SWS

Tutorial for 0110300 (Finite Element Methods) — **Practice (Ü)** — Maier, Roland

Wed 15:45-17:15 20.30 SR 2.059 from 10/23 until 02/12

0107800
4 SWS
English

Numerical methods in mathematical finance — **Lecture (V)** — Jahnke, Tobias

Thu 08:00-09:30 20.30 SR 3.061 from 10/24 until 02/13
Fri 09:45-11:15 20.30 SR 3.069 from 10/25 until 02/14

Content

An option is a contract which gives its owner the right to buy or sell an underlying asset at a future time at a fixed price. The underlying asset is typically a stock of a company or a commodity, and since its value varies randomly, computing the fair price of the corresponding option is an important and interesting problem which yields a number of mathematical challenges. This lecture provides an introduction to a number of mathematical models for option pricing. The main goal, however, is the construction and analysis of numerical methods which approximate the solution of the corresponding differential equations in a stable, accurate and efficient way. The following topics will be treated:

* Options, arbitrage and other basic concepts
* Black-Scholes equation und Black-Scholes formulas
* Numerical methods for stochastic differential equations
* (Multilevel) Monte Carlo methods
* (Quasi-)Monte Carlo integration
* Numerical methods for Black-Scholes equations
* Numerical methods for American options

Prerequisites: Participants are expected to be familiar with stochastic differential equations, the Ito integral, and the Ito formula. A short introduction to these topics (approx. 25 pages) is provided for those students who wish to acquire the relevant background through self-study. Moreover, programming skills (MATLAB or Python) are strongly recommended for the programming exercises.

0107900
2 SWS
English

Tutorial for 0107800 (numerical methods for mathematical finance) — **Practice (Ü)** — Jahnke, Tobias; Kirn, Michael

Mon 09:45-11:15 20.30 SR 3.061 from 10/21 until 02/10

0112600 4 SWS	**Numerical Methods for Integral Equations** Mon 14:00-15:30 20.30 SR 3.068 from 10/21 until 02/10 Wed 11:30-13:00 20.30 Seminarraum -1.011 (UG) from 10/23 until 02/12 **Content** In this course, we will learn about a number of methods to numerically solve integral equations, such as Nyström, collocation and Galerkin methods. The lectures will be accompanied by a programming practical in which the methods will be implemented and tested.	**Lecture (V)** Arens, Tilo
0112610 2 SWS	**Tutorial for 0112600 (Numerical Methods for Integral Equations)** Thu 11:30-13:00 20.30 Seminarraum 0.019 from 10/24 until 02/13	**Practice (Ü)** Arens, Tilo
0155450 2 SWS English	**Introduction to Kinetic Theory** Blended (On-Site/Online) Fri 14:00-15:30 20.30 SR 2.067 from 10/25 until 02/14	**Lecture (V)** Frank, Martin

Organizational topics

The course will be offered in flipped classroom format. Flipped classroom means that the lectures will be made available as videos. We will regularly meet for tutorials and discussion sessions.

Content

Kinetic descriptions play an important role in a variety of physical, biological, and even social applications, for instance, in the description of gases, radiations, bacteria or financial markets. Typically, these systems are described locally not by a finite set of variables but instead by a probability density describing the distribution of a microscopic state. Its evolution is typically given by an integro-differential equation. Unfortunately, the large phase space associated with the kinetic description has made simulations impractical in most settings in the past. However, recent advances in computer resources, reduced-order modeling and numerical algorithms are making accurate approximations of kinetic models more tractable, and this trend is expected to continue in the future. On the theoretical mathematical side, two rather recent Fields medals (Pierre-Louis Lions 1994, Cédric Villani 2010) also indicate the continuing interest in this field, which was already the subject of Hilbert's sixth out of the 23 problems presented at the World Congress of Mathematicians in 1900.

This course gives an introduction to kinetic theory. Our purpose is to discuss the mathematical passage from a microscopic description of a system of particles, via a probabilistic description to a macroscopic view. This is done in a complete way for the linear case of particles that are interacting with a background medium. The nonlinear case of pairwise interacting particles is treated on a more phenomenological level.

An extremely broad range of mathematical techniques is used in this course. Besides mathematical modeling, we make use of statistics and probability theory, ordinary differential equations, hyperbolic partial differential equations, integral equations (and thus functional analysis) and infinite-dimensional optimization. Among the astonishing discoveries of kinetic theory are the statistical interpretation of the Second Law of Thermodynamics, induced by the Boltzmann-Grad limit, and the result that the macroscopic equations describing fluid motion (namely the Euler and Navier-Stokes equations) can be inferred from abstract geometrical properties of integral scattering operators.

Link

https://www.scc.kit.edu/personen/martin.frank.php

0155460 1 SWS	**Tutorial for 0155450 (Introduction to Kinetic Theory)**	**Practice (Ü)** Frank, Martin
0100055 3 SWS English	**Parallel Computing** Tue 09:45-11:15 20.30 SR 3.061 from 10/22 until 02/11 **Content**	**Lecture (V)** Krause, Mathias Simonis, Stephan

- Parallel programming models
- Parallel solving of linear systems of equations
- Parallel finite differences, finite elements, finite volumes
- Methods of domain decomposition
- Matrix-matrix and matrix-vector operations
- Convergence and cost analysis
- Load balancing
- Applications from the natural and engineering sciences

0100057
1 SWS
English

Exercise for 0100055 (Parallel Computing) **Practice (Ü)**
Wed 14:00-15:30 20.30 Poolraum -1.031 from 10/23 until 02/12
Krause, Mathias
Simonis, Stephan

0100027
2 SWS

Stochastic Simulation **Lecture (V)**
Wed 08:00-09:30 20.30 SR 2.066 from 10/23 until 02/12
Krumscheid, Sebastian

Content

The course covers mathematical concepts and computational tools used to analyze systems with uncertainty arising across various application domains. First, we will address stochastic modelling strategies to represent uncertainty in such systems. Then we will discuss sampling-based methods to assess uncertain system outputs via stochastic simulation techniques. The focus of this course will be on the theoretical foundations of the discussed techniques, as well as their methodological realization as efficient computational tools.

Topics covered include:

- Random variable generation
- Simulation of random processes
- Simulation of Gaussian random fields
- Monte Carlo method; output analysis
- Variance reduction techniques
- Quasi Monte Carlo methods
- Markov Chain Monte Carlo methods (Metropolis-Hasting, Gibbs sampler)

Other topics that may be addressed if time allows, such as rare event simulations, and stochastic optimization using stochastic approximation or simulated annealing.

0100028
2 SWS

Tutorial for 0100027 (Stochastic Simulation) **Practice (Ü)**
Fri 11:30-13:00 20.30 SR 2.067 from 10/25 until 02/14
Krumscheid, Sebastian

0105600
4 SWS
English

Spatial Stochastics **Lecture (V)**
Tue 11:30-13:00 20.30 SR 3.069 from 10/22 until 02/11
Thu 14:00-15:30 20.30 SR 3.068 from 10/24 until 02/13
Hug, Daniel

Content

Competence Goal:
The students are familiar with some basic spatial stochastic processes. They do not only understand how to deal with general properties of distributions, but also know how to describe and apply specific models (Poisson process, Gaussian random fields). They know how to work self-organised and self-reflexive.

Content:
Random sets
Point processes
Random measures
Palm distributions
Random fields
Gaussian fields
Spectral theory of random fields
Spatial ergodic theorem

Literature

Skriptum/Lectures Notes

0105610
2 SWS
English

Tutorial for 0105600 (Spatial Stochastics) **Practice (Ü)**
Mon 15:45-17:15 20.30 SR 3.061 from 10/21 until 02/10
Hug, Daniel

0123100 2 SWS **Forecasting: Theory and Praxis** **Lecture (V)**
Tue 14:00-15:30 20.30 Seminarraum 0.016 from 10/22 until 02/11 Gneiting, Tilmann

Content

A common desire of all humankind is to make predictions for thefuture. As the future is inherently uncertain, forecasts ought to be probabilistic, i.e., they ought to take the form of probability distributions over future quantities or events. In this class, which is Part I of a two semester series, we will study the probabilistic and statistical foundations of the science of forecasting.

The goal in probabilistic forecasting is to maximize the sharpness of the predictive distributions subject to calibration, based on the information set at hand. Proper scoring rules such as the logarithmic score and the continuous ranked probability score serve to assess calibration and sharpness simultaneously, and relate to information theory and convex analysis. As a special case, consistent scoring functions provide decision-theoretically coherent tools for evaluating point forecasts. Throughout, concepts and methodologies will be illustrated in data examples and case studies.

0123110 2 SWS **Tutorial for 0123100 (Forecasting: Theory and Praxis)** **Practice (Ü)**
Fri 14:00-15:30 20.30 SR 2.058 from 10/25 until 02/14 Gneiting, Tilmann

0100009 4 SWS English **Algebraic Topology** **Lecture (V)**
Mon 11:30-13:00 20.40 Neuer Hörsaal Architektur from 10/21 until 02/10 Krannich, Manuel
Thu 15:45-17:15 20.30 Seminarraum -1.025 (UG) from 10/24 until 02/13

Content

This course provides an introduction to algebraic topology which is a fundamental discipline with ties to many areas in mathematics. Roughly, algebraic topology can be described as the study of topological spaces by means of algebraic invariants. An exemplary application is Brouwer’s fixed point theorem: Every continuous self-map from the closed unit disc in d-dimensional Euclidean space has a fixed point. The proof relies on an algebraic invariant of topological spaces called singular homology which will be the main focus of this course. Further topics include basics of category theory and homological algebra, CW complexes and cellular homology.

0100011 2 SWS **Tutorial for 010009 (algebraic topology)** **Practice (Ü)**
Wed 14:00-15:30 20.30 Seminarraum -1.013 (UG) from 10/23 until 02/12 Krannich, Manuel

0124060 2 SWS English **Extremal Graph Theory** **Lecture (V)**
Thu 09:45-11:15 20.30 SR 2.067 from 10/24 until 02/13 Aksenovich, Maria

Content

The extremal function ex(n,H) for a graph H is the largest number of edges in a graph on n vertices that does not contain H as a subgraph. The Ramsey function r(H) for a graph H is the smallest integer n such that in any 2-coloring of the edges of a complete graph on n vertices there is a monochromatic copy of H.

In this course the properties of these functions and their generalizations are considered. In addition, other extremal results are discussed. Specific topics include: Turan's theorem, classical theorems on matchings and cycles, Szemeredi's Regularity lemma and applications, Erdös-Stone-Simonovits' theorem, extremal numbers for hypergraphs, hypergraph Ramsey numbers, Ramsey numbers for graphs of bounded maximum degree and arrangeable graphs, size and online Ramsey numbers.

0124070 1 SWS English **Tutorial for 0124060 (Extremal Graph Theory)** **Practice (Ü)**
Mon 08:00-09:30 20.30 SR 2.059 from 10/21 until 02/10 Aksenovich, Maria

0100058 | **Complex Geometry** | **Lecture (V)**
3 SWS | Fri 08:00-09:30 20.30 SR 2.066 from 10/25 until 02/14 | Llosa Isenrich, Claudio

Content

This lecture will give an introduction to complex geometry. Complex geometry is the study of smooth manifolds that are locally modelled on n-dimensional complex space and one of the main goals of complex geometry is to understand the topology of complex manifolds (usually) under some additional constraints. This is a classical topic that has its origins in problems like understanding the topology of smooth zero sets of polynomials in several variables in complex projective space.

After a short introduction to complex analysis in several variables, we will introduce the most important notions, such as complex manifolds, complex vector bundles and forms. We will then focus on the important class of compact Kähler manifolds, which includes the smooth zero sets of homogeneous polynomials in complex projective space and proceed to discuss the Hodge Decomposition Theorem for their cohomology. The latter is a fundamental result in complex geometry, which places strong constraints on the topology of compact Kähler manifolds.

0100059 | **Tutorial for 0100058 (complex Geometry)** | **Practice (Ü)**
1 SWS | Thu 11:30-13:00 20.30 SR 2.059 from 10/24 until 02/13 | Llosa Isenrich, Claudio

0100049 | **Seminar (Numerical and Statistical Aspects of Machine Learning)** | **Seminar (S)**
2 SWS | Fri 11:30-13:00 20.30 SR 3.068 from 10/25 until 02/14 | Trabs, Mathias; Krumscheid, Sebastian; Frank, Martin

Content

Machine learning is ubiquitous in modern science. Its mathematical foundations lie at the intersection of optimization/optimal control, approximation theory, and statistics. To shed some light on this interplay, in this seminar we study several aspects and methods of Machine Learning from both numerical and statistical perspectives. The seminar is jointly organized by Martin Frank (SCC), Sebastian Krumscheid (SCC) and Mathias Trabs (STOCH).

0125300 | **Seminar (Modeling, Algorithms, Simulation)** | **Seminar (S)**
2 SWS | **Organizational topics** | Frank, Martin
English | Blockseminar am Ende der Vorlesungszeit

Content

Methods of computational science are essential for modern research and development in science, technology and economy. Numerous examples can be found in the fields of fluid mechanics, climate research and materials research. The mathematical modeling and algorithms used here are the focus of this seminar. The practical relevance and the optimal use of the models and algorithms and, in particular, the applications of these models and algorithms in computer simulations for concrete scientific problems, are especially emphasized. On the one hand, the optimal choice and further development of models and algorithms can significantly increase the efficiency, accuracy and reliability of simulations on high- performance computers. On the other hand, the solution of completely new scientific tasks is often based on already known models and algorithms. The topics offered for the seminar are based on current research and include mathematical-physical modeling, numerical and non-numerical algorithms as well as other methodological areas of scientific computing.

On selected topics, the students investigate current approaches to modeling and algorithms used in modern computer simulation. Students learn about practical applications of different models and numerical and non-numerical algorithms in research and development. The students deepen their knowledge in the assigned topic and critically examine literature sources and existing implementations. In doing so they try to reconstruct key points of a concrete work (a paper or a simulation code).

0120600 **Seminar** **Seminar (S)**
2 SWS Tue 09:45-11:15 20.30 Seminarraum -1.017 (UG) from 10/22 until 02/11 Liao, Xian

Content

Free interfaces between two different phases are commonly observed in everyday life. Mathematically, the free interface problem consists of one or more partial differential equations as well as an unknown, moving boundary which is to be determined.

Prüss and Simonett's book is an excellent reference for the related mathematical theory. In this seminar, we will primarily focus on the modeling and problem statements in Chapter 1 of the reference. If there are more participants, we will delve deeper into two-phase Navier-Stokes problems.

List of Topics:

1. (19.11.24) First principles in the bulk. (A, Section 1.1, Subsection 1.1)
2. (26.11.24) First principles on the interface and the dynamic problem. (A, Section 1.1, Subsections 1.2, 1.3, 1.4)
3. (03.12.24) Entropy and equilibria. (A, Section 1.2, Subsections 2.1, 2.2, 2.3, 2.4)
4. (10.12.24) The model problems. (A, Section1.3, Subsection 3.1)
5. (17.12.24) Transformation to a fixed domain. (A, Section1.3, Subsection 3.2)
6. Possible topics in January 2025: Maximal Lp-regularity, linear theory, Fixed point argument for nonlinear problem.

Prerequisites: Analysis I-III.

Reference A: J.Prüss and G. Simonett, "Moving Interfaces and Quasilinear Parabolic Evolution Equations", 2016, Springer. https://link.springer.com/book/10.1007/978-3-319-27698-4